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VISCOUS LIQUID THREE DIMENSIONAL FLOW THROUGH THE TURBO-MACHINES LABYRINTHS

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FLUXURI TRIDIMENSIONALE DE LICHIDE VÂSCOASE PRIN LABIRINTURILE TURBO-MAȘINILOR

In this paper we shall present such the numerical solving as the experimental test of the viscous liquid three dimensional flow through the clearance with and without baffle plate of the hydraulic turbo machine labyrinths, considering the ring curvature by using the flow equations written in the cylindrical trihedral coordinates.

We present also the stability of the numerical solution and the specific boundary conditions to the posed problem, as well as the installation to visualize the appearance of Sir Geoffrey Taylor's whirls.

Cuvinte cheie: debit tridimensional, fluxurile de lichide vâscoase, fluxuri hidraulice prin labirinturi/strângeri ale turbo-mașinilor

Keywords: three-dimensional flow, viscous liquid flows, flow through the hydraulic turbo machine tightening labyrinths

1. Anterior researches concerning the 3dimensional flow through the hydraulic turbo machine labyrinths

In an anterior work [1] the first of the authors effectuated, such a numerical solving of the three-dimensional flow of the viscous liquid through the labyrinths with and without baffle plate of the hydraulic turbo machines, as well as an experimental research of their flows, using besides the laminar flow visualisations through upright labyrinth with and without baffle plate, obtained in the installation for laminar

flows study [2] and two machines to put into the evidence the increase of the tightening capacity of the upright labyrinth with the rotation velocity of the mobile ring.

In the case of the three-dimensional flow theoretical study, due to the clearance smallest of labyrinth tightening with respect to the ring radius, we neglected the effect of the labyrinth rings curvature and therefore of the centrifugal forces considering the Cartesian trihedron of the motion, the numerical solving being effectuated in two successive phases.

2. The three-dimensional flow equations in cylindrical coordinates

The new hypotheses in which we have considerate the flow equations and of incompressible fluid mass conservation, are:

- the flow has a permanent character, therefore $\partial/\partial t = 0$,
- the motion has an axial symmetry, therefore $\partial/\partial\theta = 0$,
- we neglected the masses heavy forces in comparison with that of pressure, friction and inertial, which predominate in this phenomenon of liquid flow.

The Navier and Stokes equation system [3], written in cylindrical coordinates consist in the considerate hypotheses:

- three permanent motion equations, suitable to the heavy and viscous liquid

$$V_R \frac{\partial V_R}{\partial R} + V_Z \frac{\partial V_R}{\partial Z} - \frac{V_\theta^2}{R} + \frac{1}{\rho} \frac{\partial P}{\partial R} = \nu \left(\frac{\partial^2 V_R}{\partial R^2} + \frac{1}{R} \frac{\partial V_R}{\partial R} + \frac{\partial^2 V_R}{\partial Z^2} - \frac{V_R}{R^2} \right), \quad (1)$$

$$V_R \frac{\partial V_\theta}{\partial R} + V_Z \frac{\partial V_\theta}{\partial Z} + \frac{V_R V_\theta}{R} = \nu \left(\frac{\partial^2 V_\theta}{\partial R^2} + \frac{1}{R} \frac{\partial V_\theta}{\partial R} + \frac{\partial^2 V_\theta}{\partial Z^2} - \frac{V_\theta}{R^2} \right), \quad (2)$$

$$V_R \frac{\partial V_Z}{\partial R} + V_Z \frac{\partial V_Z}{\partial Z} + \frac{1}{\rho} \frac{\partial P}{\partial Z} = \nu \left(\frac{\partial^2 V_Z}{\partial R^2} + \frac{1}{R} \frac{\partial V_Z}{\partial R} + \frac{\partial^2 V_Z}{\partial Z^2} \right), \quad (3)$$

- and the incompressible fluid mass conservation

$$\frac{1}{R} \frac{\partial (R V_R)}{\partial R} + \frac{\partial V_Z}{\partial Z} = \frac{V_R}{R} + \frac{\partial V_R}{\partial R} + \frac{\partial V_Z}{\partial Z} = 0, \quad (4)$$

the reference drawing of the upright and with baffle labyrinth being represented in the figure 1,a and b

2.1. Dimensionless form of the equation system with partial differential

For the generalization of the numerical solving we shall choose the dimensionless variables and functions:

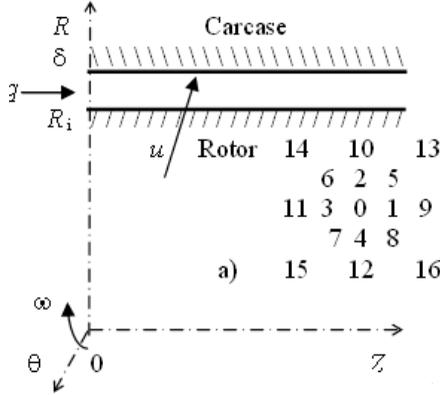


Fig. 1 Upright labyrinth

$$z = \frac{Z}{R_i}, \quad r = \frac{R}{R_i}, \quad v = \frac{V_R}{U_i}, \quad u = \frac{V_0}{U_i}, \quad w = \frac{V_Z}{U_i}, \quad p = \frac{P}{P_i}, \quad \psi = \frac{\Psi}{R_i U_i}, \quad (5)$$

in which case the equation system becomes:

$$v'_r v + v'_z w - \frac{u^2}{r} + \text{Eu } p'_r = \frac{1}{\text{Re}} \left(v''_{r^2} + \frac{1}{r} v'_r + v''_{z^2} - \frac{v}{r^2} \right), \quad (1')$$

$$u'_r v + u'_z w + \frac{v u}{r} = \frac{1}{\text{Re}} \left(u''_{r^2} + \frac{1}{r} u'_r + u''_{z^2} - \frac{u}{r^2} \right), \quad (2')$$

$$w'_r v + w'_z w + \text{Eu } p'_z = \frac{1}{\text{Re}} \left(w''_{r^2} + \frac{1}{r} w'_r + w''_{z^2} \right), \quad (3')$$

$$v'_r + \frac{v}{r} + w'_z = \mathbf{0}, \quad (4')$$

where we notated with: $Eu = P_1/\rho U_1^2$ – the Euler number and $Re = R_1 U_1/\nu$ – the Reynolds number.

2.2. Elimination of the pressure function

The pressure function being unknown on the all domain boundaries of the fluid flow, we shall eliminate in the virtue of Schwarz's commutative relation $p''_{rz} = p''_{zr}$ of the mixed partial differential of 2nd order, obtaining by subtraction of the both partial differentials, the relation

$$v''_{r^2z} + v''_{z^3} - w''_{r^2z} - w''_{z^2r} + \frac{1}{r} v''_{rz} - \frac{1}{r} w''_{r^2} - \frac{1}{r^2} v'_z + \frac{1}{r^2} w'_r - \text{Re} \left(v''_{rz} v + v'_r v'_z + v''_{z^2} w + v'_z w'_z - \frac{2}{r} u'_z u - w''_{r^2} v - w'_r v'_r - w''_{rz} w - w'_z w'_r \right) = 0. \quad (6)$$

2.3. The introduction of the streamline function in the motion meridian plane

To eliminate the fluid mass conservation equation, unstable in the iterative numerical calculus, we shall introduce the streamline function by the relations:

$$v = \frac{1}{r} \psi'_z, \quad \text{and} \quad w = -\frac{1}{r} \psi'_r \quad (7)$$

and calculating the partial differentials, which intervene in relations (6) and (2'), we shall obtain the partial differential equation of the streamline function

$$\psi''_{r^4} + \psi''_{z^4} + 2\psi''_{r^2z^2} - \frac{2}{r} \psi''_{r^3} - \frac{2}{r} \psi''_{rz^2} + \frac{3}{r^2} \psi''_{r^2} - \frac{3}{r^3} \psi'_r - \text{Re} \left[\frac{1}{r} \psi'_z \left(\psi''_{rz^2} + \psi''_{r^3} - \frac{3}{r} \psi''_{r^2} - \frac{2}{r} \psi''_{z^2} \right) - \frac{1}{r} \psi'_r \left(\psi''_{z^3} + \psi''_{r^2z} - \frac{1}{r} \psi''_{rz} - \frac{3}{r^2} \psi'_z \right) - 2u'_z u \right] = 0 \quad (8)$$

and for the tangential velocity component

$$\text{Re} \left(\frac{1}{r} \psi'_z u'_r - \frac{1}{r} \psi'_r u'_z + \frac{u}{r^2} \psi'_z \right) = u''_{r^2} + \frac{1}{r} u'_r + u''_{z^2} - \frac{u}{r^2}. \quad (2'')$$

3. The numerical solving method of the three-dimensional flow

Developing in Taylor series [2] on the network constant step $\chi = \delta z = \delta r$ the two functions: of the tangential velocity component $u(z, r)$ till the 2nd order:

$$u_{1,3} = u_0 \pm \chi u'_z + \frac{\chi^2}{2} u''_{z^2} \pm \dots \quad u_{2,4} = u_0 \pm \chi u'_r + \frac{\chi^2}{2} u''_{r^2} \pm \dots, \quad (9)$$

from which we can calculate the partial differential expressions:

$$u'_{z,r} = \frac{u_{1,2} - u_{3,4}}{2\chi} \quad u''_{z^2,r^2} = \frac{1}{\chi^2} (u_{1,2} - 2u_0 + u_{3,4}), \quad (10)$$

and of the streamline function $\psi(z, r)$ till the 4th order, from which we shall calculate the partial differential expressions: (11)

$$\psi'_{z,r} = \frac{1}{\chi} \left[\frac{2}{3} (\psi_{1,2} - \psi_{3,4}) + \frac{1}{12} (\psi_{11,12} - \psi_{9,10}) \right], \quad \psi''_{z^3,r^3} = \frac{1}{\chi^3} \left[\frac{2}{3} (\psi_{9,10} - \psi_{11,12}) + \frac{4}{3} (\psi_{3,4} - \psi_{1,2}) \right]$$

$$\psi''_{z^2,r^2} = \frac{1}{\chi^2} \left[\frac{4}{3} (\psi_{1,2} + \psi_{3,4}) - \frac{5}{2} \psi_0 - \frac{1}{12} (\psi_{9,10} + \psi_{11,12}) \right], \quad (12)$$

$$\psi''_{zr} = \frac{1}{\chi^2} \left[\frac{1}{3} (\psi_5 + \psi_7 - \psi_6 - \psi_8) - \frac{1}{48} (\psi_{13} + \psi_{15} - \psi_{14} - \psi_{16}) \right], \quad (13)$$

$$\psi'''_{zr^2} = \frac{1}{\chi^3} \left[\frac{2}{3} (\psi_5 - \psi_6 - \psi_7 + \psi_8) + \frac{4}{3} (\psi_3 - \psi_1) \right], \quad (14)$$

$$\psi'''_{z^2r} = \frac{1}{\chi^3} \left[\frac{2}{3} (\psi_5 + \psi_6 - \psi_7 - \psi_8) + \frac{4}{3} (\psi_4 - \psi_2) \right], \quad (15)$$

$$\psi^{IV}_{z^4,r^4} = \frac{1}{\chi^4} \left[16\psi_0 - \frac{32}{3} (\psi_{1,2} + \psi_{3,4}) + \frac{8}{3} (\psi_{9,10} + \psi_{11,12}) \right], \quad (16)$$

$$\psi^{IV}_{z^2r^2} = \frac{1}{\chi^4} \left[\frac{8}{3} \sum_5^8 \psi_i + \frac{32}{3} \psi_0 - \frac{16}{3} \sum_1^4 \psi_i \right]. \quad (17)$$

4. The algebraic relations associated to the partial differential equations

Introducing the partial differential expressions (11) ÷ (17) in the equations (2') and (8), we shall obtain the algebraic relations

associated to the partial differential equations: of the peripheral velocity component

$$\begin{aligned}
 u_0 &= \frac{1}{\frac{4}{\chi^2} + \frac{1}{r_0^2}} \left\{ -\text{Re} \left\{ \frac{1}{r_0 \chi} \left[\frac{2}{3} (\psi_1 - \psi_3) + \frac{1}{12} (\psi_{11} - \psi_9) \right] \left(\frac{u_2 - u_4}{2\chi} + \frac{u_0}{r_0} \right) - \left[-\frac{1}{r_0 \chi} \left[\frac{2}{3} (\psi_2 - \psi_4) + \frac{1}{12} (\psi_{12} - \psi_{10}) \right] \frac{u_1 - u_3}{2\chi} \right] \right\} \right\} = \\
 &= \frac{1}{4 + \frac{\chi^2}{r_0^2}} \left\{ \sum_1^4 u_i + \frac{\chi}{2r_0} (u_2 - u_4) - \left[-\text{Re} \chi \left\{ v_0 \left(\frac{u_2 - u_4}{2} + \frac{u_0 \chi}{r_0} \right) + w_0 \frac{u_1 - u_3}{2} \right\} \right] \right\}, \quad (2'')
 \end{aligned}$$

as well as of the streamline function in the meridian flow plane of the simplified form necessary to the numerical stability study, in which we shall considered that $r_0 \cong 1$ and the grid step $\chi \cong 0.1$

$$\begin{aligned}
 & \left\{ \begin{aligned} & 64 \sum_1^4 \psi_i - 16 \sum_5^8 \psi_i - 8 \sum_9^{12} \psi_i + \quad (10') \\ & + \frac{2\chi}{r_0} \left[8(\psi_4 - \psi_2) + 2(\psi_5 + \psi_6 - \psi_7 - \psi_8 + \psi_{10} - \psi_{12}) \right] - \\ & - \frac{3\chi^2}{r_0^2} \left[4(\psi_2 + \psi_4) - \frac{1}{4}(\psi_{10} + \psi_{12}) \right] + \frac{3\chi^3}{r_0^3} \left[2(\psi_2 - \psi_4) + \frac{1}{4}(\psi_{12} - \psi_{10}) \right] + \\ & v_0 \left\{ \begin{aligned} & 2(\psi_5 + \psi_6 - \psi_7 - \psi_8 + \psi_{10} - \psi_{12}) + 8(\psi_4 - \psi_2) - \\ & - \frac{3\chi}{r_0} \left[4(\psi_2 + \psi_4) - \frac{1}{4}(\psi_{10} + \psi_{12}) - \frac{15}{2} \psi_0 \right] - \\ & - \frac{2\chi}{r_0} \left[4(\psi_1 + \psi_3) - \frac{15}{2} \psi_0 - \frac{1}{4}(\psi_9 + \psi_{11}) \right] - \end{aligned} \right\} + \\ & + \text{Re} \chi \left\{ \begin{aligned} & 2(\psi_5 - \psi_6 - \psi_7 + \psi_8 + \psi_9 - \psi_{11}) + 8(\psi_3 - \psi_1) - \\ & - \frac{\chi}{r_0} \left[(\psi_5 + \psi_7 - \psi_6 - \psi_8) - \frac{1}{16}(\psi_{13} + \psi_{15} - \psi_{14} - \psi_{16}) \right] - \\ & - \frac{3\chi^2}{r_0^2} \left[2(\psi_1 - \psi_3) + \frac{1}{4}(\psi_{11} - \psi_9) \right] \end{aligned} \right\} - \\ & - 3u_0 \chi^2 (u_1 - u_3) \end{aligned} \right\}
 \end{aligned}$$

$$\delta u_{n+1}^{\pm z} \cong \left(\frac{1}{4} \pm \frac{\text{Re } w_0 \chi}{8} \right) \delta u_n, \quad \delta u_{n+1}^{\pm r} \cong \left(\frac{1}{4} \mp \frac{\chi}{8r_0} \pm \frac{\text{Re } v_0 \chi}{8} \right) \delta u_n, \quad (18)$$

5. The numerical solution stability study

For the rotational fluid velocity component from the formula (2'') we shall calculate the error relaxation [2] on the two directions, considering that the initial its value is equal with unity $\delta u_i = 1$, as function of the calculus iteration and for different local Reynolds number $\text{Re } v_0 \chi$ respectively $\text{Re } w_0 \chi$, the error relaxation diagram of δu_n for instance on the grid direction $\pm r$ being represented in the figure number 2.

For the streamline function we have represented in the figure 3 the error relaxation diagram of $\delta \psi_n$ for instance on the grid direction $\pm z$ for different local Reynolds number Rw , considering also its initial value equal with unity $\delta \psi_i = 1$, the error relaxation formula having the following form (19)

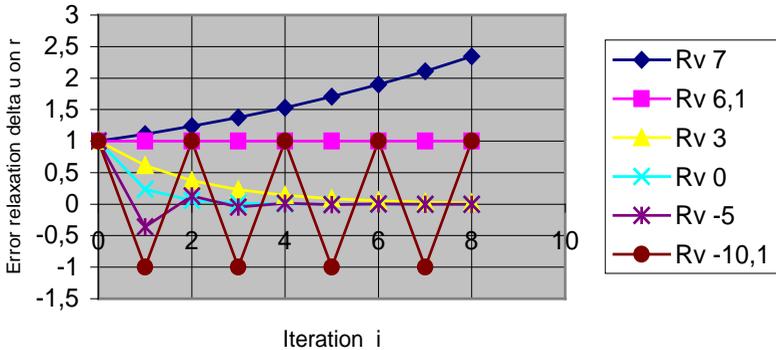


Fig. 2 The δu error relaxation diagram for different local Reynolds number Rv

$$\delta \psi_{n+1}^{\pm z} \cong \left(\frac{1}{4} \pm \frac{\text{Re } w_0 \chi}{8} \right) \delta \psi_n, \quad \delta \psi_{n+1}^{\pm r} \cong \left(\frac{1}{4} \mp \frac{\chi}{8r_0} \pm \frac{\text{Re } v_0 \chi}{8} \right) \delta \psi_n, \quad (19)$$

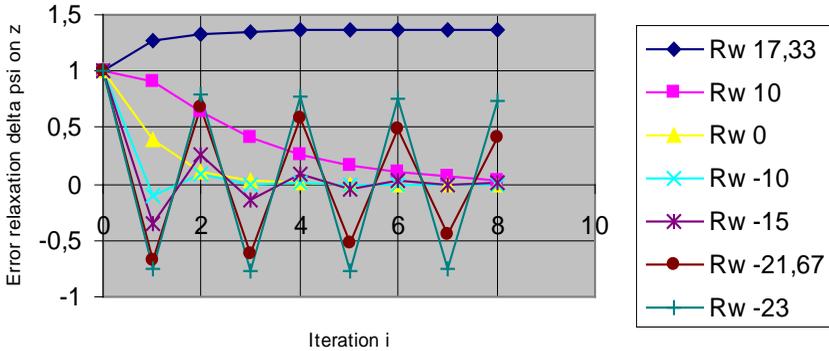


Fig. 3 The $\delta\psi$ error relaxation diagram for different local Reynolds number Rw

6. The initial and boundary conditions

In the figure 4 we represented the boundary conditions, but the **initial arbitrary conditions** being the linear variation of the velocity component $u(z,r)=11-10r$ and for the streamline function we can take any values different of zero, for instance

$$\psi_i(z > 0, r) \approx 0,1$$

to avoid the banal solution, dividing for instance the wide in 10 radial portions between the interior radius $r_i = 1$ and this exterior $r_e = 1,1$ obtaining the grid relative step

$$\chi = (r_e - r_i)/10 = 0,01.$$

The values of the two functions being considerate the fixed values as the **boundary conditions** on the two clearance rings

$$u_i(z, r_i) = 1 \quad \text{as well} \quad u_e(z, r_e) = \psi(z, r_i) = \psi(z, r_e) = \psi(0, r) = 0, \quad (20)$$

As **boundary conditions** we considered the reflection condition [1] [2] for the stream line function even with respect to the mobile ring which rotates on the other direction, as well as for the both functions the **flow symmetry** with respect to the axis $z = 0$.

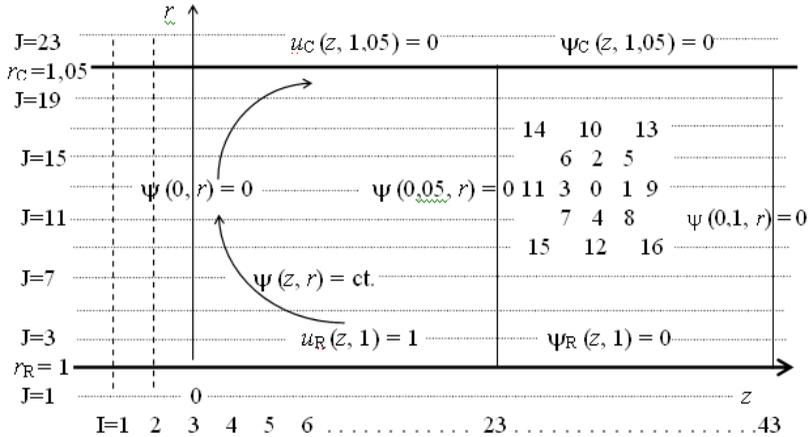


Fig. 4 The calculus domain with the knot numbering and the boundary conditions

The manner to run by calculus the domain will be in the sense $+r$ and from $z = 0$ to the values $z > 0$, the lines being notate from $J = 1$ to $J = 23$, the two exterior lines of the domain being necessary for the streamline function ψ and the columns begin of the $l = 1, 2, \dots, 23, \dots, 43$ the first column being necessary to the flow symmetry condition with respect to the right $z = 0$.

7. The elaboration of the calculus programme

The calculus programme will comprise the algebraic relations associated to the two functions $u(2'')$ and $\psi(10')$, their arbitrary values for the calculus beginning, as well as the boundary conditions specific to the two functions, the calculus begin for the velocity function u from the knot $l = 3$ and $J = 3$.

For the **peripheral velocity component** we shall have the following **boundary conditions**.

8. The obtained resultants

One presents the obtained results by numerical integration of Navier-Stokes equations of viscous liquid motion, concerning the appearance of the Sir Geoffrey Taylor's whirls as in the figure number 5, due to the interior cylinder rotation, the exterior cylinder being fixed.

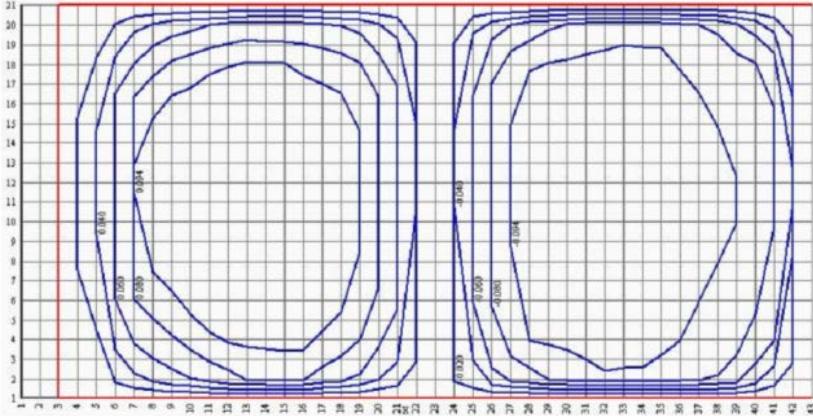


Fig. 5 The schematic representation of two Sir Geoffrey Taylor's whirls of contrary rotations between the two cylinders: the exterior being fixed and the interior mobile

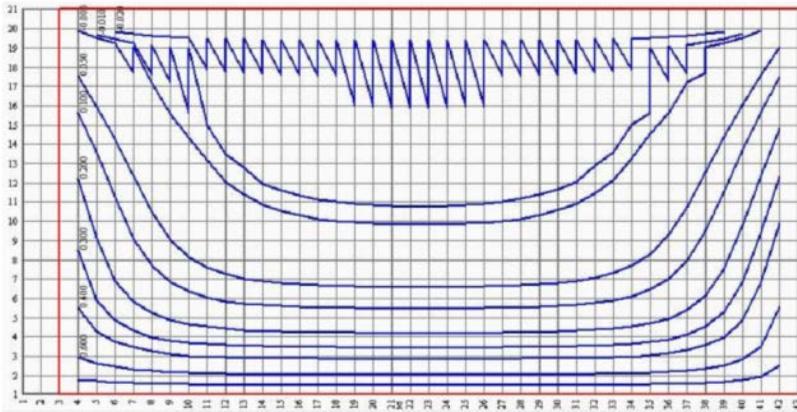


Fig. 6 One presents the schematic representations of constant liquid rotational velocity u component, induced by the rotational motion of the interior cylinder in the domain of the two Sir Geoffrey Taylor's whirls

In the figure 6 one can see the schematic representations of the constant liquid rotational velocity u component, induced by the rotational motion of the interior cylinder in the domain of the two Sir Geoffrey Taylor's whirls.



Fig. 7 The experimental installation, constituted as in the figure 1, from a cylindrical body driven with variable rotational velocity by coupling with an electric motor from continuum current, to be possible to visualise the Sir Geoffrey Taylor's whirls

9. Experimental researches

In the figure 7 we presented the Sir Geoffrey Taylor's whirls obtained on an experimental installation in our laboratory of Pumps, Ventilators, Blowers and Compressors of the Polytechnic University of Bucharest, Department of Hydraulics, Hydraulic Machines and Environment Engineering.

The experimental stand is represented in the figure 7.

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FLUXURI TRIDIMENSIONALE DE LICHIDE VÂSCOASE PRIN LABIRINTURILE TURBO-MAȘINILOR

Rezumat: În lucrare se prezintă rezolvarea numerică ca încercare
experimentală a fluxului de lichid vâscos în trei dimensiuni prin clearance-ul, cu
și fără placă de diafragmă din labirinturi de mașini hidraulice turbo, având în
vedere inelul de curbură cu ajutorul ecuațiilor de debit scrise în triedru cilindric.
Se prezintă, de asemenea, stabilitatea soluției numerice și condițiile specifice
de frontieră la problema ridicată, precum și de instalare pentru a vizualiza
aspectul de vârtejuri a lui Sir Geoffrey Taylor.