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„Profesorul Dorin Pavel – fondatorul hidroenergeticii românești”
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DESPRE O METODĂ GRAFICĂ DE DETERMINARE A CONTURULUI OPTIM PENTRU ÎNVELITOAREA CILINDRU-SFERĂ SUB PRESIUNE INTERNĂ

Dora FLOREA

ABOUT A GRAPHICAL METHOD OF OPTIM CONTOUR DETERMINATION FOR THE SPHERE-CYLINDER SHELL UNDER INTERNAL PRESSURE

The study of the analytical method for determination an optim contour for the vessels sphere-cylinder under internal pressure [1] with the achievement a uniform state of the stress, regarding the stress state from vessel, it considered the analytic equations for the stress state in the shell.

Present study propose to solve the same problem but used the graphics of equivalents Von Mises stress used the approximation method of the graphics through the spline functions.

Studiul metodei analitice de determinare a unui contur optim pentru invelitoarea cilindru-sferă sub presiune internă [1] cu scopul obținerii unei stări de tensiuni uniforme, privitor la starea de tensiuni din recipient, consideră ecuațiile analitice pentru starea de tensiuni în învelitoare.

Prezentul studiu propune rezolvarea problemei utilizând graficele tensiunilor echivalente von Mises utilizând metoda de aproximare a graficelor prin funcții spline.

Keywords: Graphics of Echivalente von Mises stress, sphere-cylinder, internal pressure, spline functions

Cuvinte cheie: grafica stresului echivalente von Mises, sferă-cilindru, presiune internă, funcții spline

1. İntroduction

The analytical method for the optimization of the exterior contour of a sphere-cylinder vessel under internal uniform axial symmetrical pressure, presented in the paper [1], used for the start the analytical equations what defined the stress state in the shell. Because of many times not know the analytical equations of the stress state , so the stress state in the thick shell for the sphere-cylinder vessel, in this study propose the optimization the contour shell, start from the graphics of equivalents Von Mises stress acquired through the FEM method, results after the run of element finites programme NASTRAN V4.0 [3].

The input values are the equivalent von Mises stress functions $y_{i,i=1...n}$ for the n equidistant points $x_{i,i=1...n}$ on the shell, for the various thickness of the shell $g_{k,k=1...m}$.

$$F_k(x_i,g_k)=y_i \quad , \quad i = 1..n , k=1..m \tag{1}$$

Because to impose the knowledge in the final of the thickness values of the shell $t_{i,i=1...n}$ in n points of the shell for a value Y_u of the equivalents von Mises stress, it impose to determine the function:

$$t_i = T(Y_u,x_i) \quad , \quad i = 1..n \tag{2}$$

2. Theoretical Considerations

The functions family $y_{i,i=1...n}$ (1) express the dependence between the equivalents von Mises stresses and the points of the shell $x_{i,i=1...n}$, determined for a finite value of the thickness parameters of the shel $g_{k,k=1...m}$ and because is necessary know the thickness of the shell $t_{i,i=1...n}$ in n points from shell in the condition keeping a constant value Y_u for equivalents von Mises stress in all n points from shell, it necessary the application an interpolation method and in this study selected the interpolation through spline functions which benefit of the remarkable propriety of minimum curvature [2].

The function of expressing of the shell thickness for which to determine $t_{i,i=1...n} = G(Y_n, x_i)$ in n points are defined on the interval [a,b] and be Δ an interval of division :

$$a = x_1 < x_2 <\dots x_n = b \tag{3}$$

For the determination of function G: [a,b]→R it necessary to know the functions family values $f_{kv}= G_{i,i=1\ldots n}(y_k,x_i)$ defined in m points k=1…m, for n points i=1…n started from the functions family $F_{k,}$ k=1…m (1).

The function G_i being a tabbing function for it determination in n points for a value of equivalents von Mises stress Y_u establishes to be constant in n points of the shell, to build a spline function for the function G_i in the end the determination the function t_i (2).

The function spline [2] of order n relative at the dimension Δ of the interval [a,b] is a function S:[a,b]→R of class C^{n-1}[a,b], with the restriction $S_i(y)$ on each subinterval $[y_k,y_{k+1}]$ of the division, are the polinome of the order j, so:

$$S_i(y)=P_j^{(i)}(y) \quad \text{if } y\epsilon[y_k,y_{k+1}],\ k=1\ldots m-1 \tag{4}$$

The spline function S(y) has the first derivate (j-1) continuous on the interval [a,b] , the derivate of j order discontinuous in the node y_k of the division, so he is a smooth function on the part, the smooth grade of the spline function is gived of the order j.

Definition It name the spline function [2] of interpolation for the function Gi , spline function S(y) on the give division which accomplish the interpolation conditions:

$$S(y_k)=g_k\ ,\ k=1\ldots m \tag{5}$$

It considers the cubic spline functions of interpolation and in this case the restriction $S_k(y)$ are the polynomial of the third order:

$$S_k(y)=A_ky^3+B_ky^2+C_ky+D_k \qquad \text{if } y\epsilon[y_k,y_{k+1}],\ k=1\ldots m-1 \tag{6}$$

The cubic spline functions are of C^2[a,b] class, so are continuous together with first two derivate. The coefficients A_k,B_k,C_k,D_k of the each restriction can be determinates from conditions of continous in the points of the division.
The second derivate of the function $S_k(y)$ is a linear function and so:

$$\frac{S_k^{//}-D_k^*}{y-y_k}=\frac{D_{k+1}^*-D_k^*}{y_{k+1}-y_k} \tag{7}$$

where

$$D_k^{*}=S^{//}(y_k)\ ,\ k=1\ldots m$$

are the values of the second derivate of the spline function in the nodes of the interpolation network, the relation (7) may be set under form:

$$S_k^{//}(y)=\frac{D_{k+1}^{*}(y-y_k)+D_k^{*}(y_{k+1}-y)}{h_k} \tag{8}$$

with

$$h_k=y_{k+1}-y_k\ ,\quad k=1\ldots m-1$$

Integrating of twice , the relation (8) became successive:

$$S_k^{/}(y)=\frac{D_{k-1}^{*}(y-y_k)^2-D_k^{*}(y_{k+1}-y)^2}{2h_k}+C_k^{/} \tag{9}$$

$$S_k(y)=\frac{D_{k-1}^{*}(y-y_i)^3+D_k^{*}(y_{k+1}-y)^3}{6h_k}+C_k^{/}y+C_k^{//} \tag{10}$$

$$\frac{D_k^{*}h\frac{2}{k}}{6}+C_k^{/}x_k+C_k^{//}=g_k \tag{11}$$

$$\frac{D_{k+1}^{*}h_k^2}{6}+C_k^{/}x_{k+1}+C_k^{//}=g_{k+1} \tag{12}$$

and the results:

$$C_k^{/} = \frac{g_{k+1} - g_k}{h_k} - \frac{(D^*_{k+1} - D^*_k)h_k}{6} \tag{13}$$

$$C^{//}{}_k = \frac{y_{k+!}g_k - y_k g_{k=1}}{h_k} + \frac{(y_k D^*{}_{k+1} - y_{k+1} D^*{}_k)h_k}{6} \tag{14}$$

Replacing these expressions in the relation (10) and identifying the coefficients of the y powers, results for the coefficients of restrictions (6) following:

$$A_k = \frac{D^*{}_{k+1} - D^*{}_k}{6h_k} \tag{15}$$

$$B_k = \frac{D_k{}^* y_{k+1} - D^*{}_{k+1}\, x_k}{2h_k} \tag{16}$$

$$C_k = \frac{D^*{}_{k+1} y^2{}_k - D^*{}_k y^2{}_{k+1}}{2h_k} + \frac{g_{k+1} - g_k}{h_k} - A_k h^2{}_k \tag{17}$$

$$D_k = \frac{D^*{}_k y^3{}_{k+1} - D^*{}_{k+1} y^3{}_k}{6h_k} + \frac{g_k y_{k+1} - g_{k+1} y_k}{h_k} - \frac{B_k h^2{}_k}{3} \tag{18}$$

For the complete defining of the spline functions must determined its derivate of two ordin D_k^* in the points of division. In this purpose it imposed the continuity of the first derivate of the spline function in this points:

$$S'_{k-1}(x_k) = S'_k(x_k) \qquad (19)$$

Used the expression (9) for the first derivate and consider expression (13) for the constants of integration, the expression (16) is;

$$\frac{h_{k-1}}{6} D^*_{k-1} + \frac{h_{k-1}+h_k}{2} D^*_k + \frac{h_k}{6} D^*_{k+1} = \frac{g_{k+1} - g_k}{h_k} - \frac{g_k - g_{k-1}}{h_{k-1}}$$

k=2,3…m-1

(20)

This is a system of (m-2) equations having the unknowns the m derivate of two order D_k^* of the spline functions in the interpolation nodes.

From the condition of the interval heads $[y_1, y_m]$, it can obtain two supplementary relations. Suppose [2] that are know the derivate g_1' and g_m' in these points, so:

$$S'_1(y_1) = g'_1$$

$$S'_{m-1}(y_m) = g'_m$$

or

(21)

$$\frac{h_1}{3} D^*_1 + \frac{h_1}{6} D^*_2 = \frac{g_2 - g_1}{h_1} - g'_1$$

$$\frac{h_{m-1}}{6} D^*_{m-1} + \frac{h_{m-1}}{3} D^*_m = g'_m \frac{g_m - g_{m-1}}{h_{m-1}}$$

with these relations it obtain the following linear equation system with the three diagonal matrice for the determination the derivate of two ordin for the spline functions:

$$b_1 D^*_1 + c_1 D^*_2 = d_1$$

(22)

$$a_j D^*_{j-1} + b_j D^*_j + c_j D^*_{j+1} = d_j \qquad a_m D^*_{m-1} + b_m D^*_m = d_m$$

where

$a_1 = 0 \quad b_1 = 2h_1 \quad c_1 = h$
$$d_1 = 6\left(\frac{g_2 - g_1}{h_1} - g'_1\right)$$

$a_j = h_{j-1}$ $b_j = 2(h_{j-1}+h_j)$ $c_j = h_j$ $j = 2,\ldots m-1$

$$d_j = 6\left(\frac{g_{j+1} - g_j}{h_j} - \frac{g_j - g_{j-1}}{h_{j-1}}\right) \tag{23}$$

$a_m=h_{m-1}$ $b_m=2h_{m-1}$ $c_m=0$
$$d_m = 6\left(g'_m - \frac{g_m - g_{m-1}}{h_{m-1}}\right)$$

The linear equation system (22) it can resolve [2] used a solving method for the equations system with the three diagonal matrices. After obtain the values D_j^* it can compute the coefficients A_j, B_j, C_j, D_j of the restrictions with the relations (15), the spline functions being defined.

3. Conclusions

To know the spline function $S(y_k) = g_{k,k=1\ldots m}$ for the tabling function G_i mean kcnowledge of the dependence between thickness and equivalente von Mises stress for the point x_i of the shell in interval determinated of the m points.

The value of the shell thickness g_i in the point i of the vessel for a imposed value of echivalente Von Mises stress $y = Y_n$ it can determined intersecting the spline function $S(y_k) = g_{k,k=1\ldots m}$ with the equivalente von Mises stress $y = Y_u$ imposed to realize in the shell.

In this way it can determine for all the points i = 1…n , the value of the shell thickness t_i and so the function $t_i = T(Y_u, x_i)_{i=1\ldots n}$ what asked.

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Dr.Ing. Dora FLOREA
Departamentul de Mecanică şi Rezistenţa materialelor,
Facultatea de Mecanică
Universitatea ”Politehnică” Timişoara
membru AGIR,
e-mail: Florea_Aglaia@yahoo.com